ISSN 0317-2473

Research Discussion Paper No. 83

Structural Equation Models of Multiple Respondent Data: Mutual Influence Versus Family Factor Approaches

by

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October, 1991

92-04419/

This paper describes two structural equation models for "multiple respondent" data.[1] In multiple respondent data, the unit of analysis, or case, consists of a set of individuals with common membership in a social network. Good examples of such data are Duncan, Haller, and Portes (1968) classic study of peer influences on aspirations, Hauser and Mossel's (1984) study of the educational and occupational attainment of brothers, and Thomson and Williams's (1982) study of the fertility plans of husbands and wives. A typical interest in the analysis of these data is explaining the relation between the variables that represent each member's response to the same or similar questions: the aspirations of peers, the occupational attainment of brothers, and the couple's respective plans for another child.

There are two approaches taken in developing models of such data: the mutual influence approach and the family factor approach. The structural equation model for the mutual influence approach treats the different reports as measures of members' characteristics and uses reciprocal paths to explain the relationships between these variables. The observed variables may be treated as fallible measures of latent variables. Even when this is done, however, each latent variable is still interpreted as a characteristic of the member rather than as a property of the social network. Duncan, et. al.'s (1968) model of peer influences provides the best known example of this type of approach. Family factor models, in contrast, treat the different reports as measures of a single latent variable which represents a property of the social network. The work of Thomson and Williams (1982) on the couple's plans for another child and the work of Hauser and his associates on the status attainment of siblings provide examples of this approach (Hauser and Mossel, 1984; Hauser, 1988; Hauser and Wong, 1989).

Although the differences between the two approaches raise interesting conceptual and theoretical issues, this paper focuses on the structural differences between the two models and on the consequences of attempting to fit one model to data generated by the structure of the other

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[1] Part of the data used in this paper were made available by the Inter-university Consortium for Political and Social Research. The data for the youth-parent socialization study, 1965, were originally collected by M. Kent Jennings; the followup data for the 1973 panel were collected by Jennings and Richard G. Niemi. Neither the original collectors of the data nor the consortium bear any responsibility for the analyses of interpretations presented here.

model. I describe these differences by comparing below the "within-member" and "cross-member" reduced-form slopes for the two models.[2] The mutual influence model implies that the reduced-form slopes for the within-member regressions will typically exceed the reduced-form slopes for the cross-member regressions. On the other hand, the family factor model implies that the relative magnitude of the reduced-form slopes is proportional to the loading of the endogenous variable on the family factor. In the case of equal loadings, the within-member and cross-member slopes for the same exogenous variable will equal one another. When the loadings differ, the ratio of the within- to cross-member slopes for one exogenous variable will equal the inverse of this ratio for the other exogenous variable.

The consequence of this difference for fitting a family factor model to data generated by a mutual influence process is straightforward. The model will not fit the data; specifying a mutual influence model will yield a fit that is significantly better and usually acceptable. Fitting a mutual influence model to data generated by a family factor structure poses a different set of problems. Because the ratio of one set of reduced-form coefficients will approximately equal the inverse of the ratio of the other set, the product of the two ratios will approach unity. This result violates the stability assumption of the mutual influence model (Joreskog and Sorbom, 1988). It poses a logical problem for the mutual influence model since it implies that the denominator of the reduced-form coefficients is approximately zero. Consequently, the analyst will have difficulty obtaining a solution, and, if one is obtained, it will be a degenerate one: (a) the estimates of both reciprocal paths will approach unity when the loadings of both endogenous variables on the family factor are approximately equal; (b) the estimates of the structural effects of the exogenous variables will approach zero; and (c) under certain conditions the variance of the disturbances for the endogenous variables will also approach zero.

I analyze two data sets to illustrate these points. The first is the matrix of correlations and

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[2] A within-member slope results from the regression of the endogenous variable(s) for a member on the exogenous variable for the same member. A cross-member slope results from the regression of the endogenous variable for a member on the exogenous variable(s) for another member of the same network.

standard deviations reported by Thomson and Williams (1982) in their paper on the effects of wives' and husbands' respective "taste" for children on their respective plans for having another child. Despite its status as an example of the family factor approach, I find that a mutual influence model achieves a fit to the data that is both acceptable and significantly better than the fit of their model which specifies a couple factor. The second data set comes from Jennings and Niemi (1971, 1981) study of political socialization in which high school seniors and one or both parents were interviewed in 1965 and then again in 1973. I find that the 1965 plans of the student, the mother, and the father for the student's post-secondary education measure a family factor that intervenes between the schooling of father and mother and the student's schooling reported in 1973.

My discussion of the results emphasizes the need to keep both models in mind when analyzing multiple respondent data or, indeed, any data in which either reciprocal effects or a latent variable could provide a plausible explanation for the correlations among a set of endogenous variables. Despite my emphasis on the difference between the two models, I conclude by suggesting that we can view the family factor model as a limiting case of the mutual influence model in which the members of the network have achieved complete consensus.

THE STRUCTURE OF THE MUTUAL INFLUENCE AND FAMILY FACTOR MODELS

This section contains a non-rigorous but detailed description of the structure of the two models. The notation I use draws heavily from Duncan (1975) but also borrows from the authors of current software for structural equation models (Bentler, 1989; Joreskog and Sorbom, 1988). To simplify the presentation, my notation glosses over the important distinction between parameters and parameter estimates. For the sake of simplicity, I also sacrifice some generality by assuming that all parameters in the models described below have positive signs.

Assume that we have four variables: x_1, x_2, y_3, and y_4. The x's are exogenous variables; the y's are endogenous variables. To make the discussion of the relations among these variables concrete, I will define them in terms of the Thomson-Williams data. Variables x_1 and x_2 stand for the husband's and wife's respective "taste" for children, while y_3 and y_4 stand for their respective

plans for another child. Note that these variables illustrate the special feature of multiple respondent data referred to above. To wit: x_1 is "the same" as x_2 and y_3 is "the same" as y_4 in the sense that both variables in a pair refer to the same content, and both use the same scale of measurement. This feature of multiple respondent data allows the researcher to make fairly strong assumptions about values of the reciprocal effects in a mutual influence model and about the loadings of the variables on the latent variables in a family factor model.

In both the Thomson-Williams and Jennings-Niemi data the exogenous variables are latent variables rather than observed directly. Each parent's taste is measured by two indicators in the case of the Thomson-Williams data, while each exogenous variable is measured by three indicators in the case of the Jennings-Niemi data. My discussion of the family factor model attends to the distinction between observed and latent values of the endogenous variables; my discussion of the mutual influence model assumes that both endogenous variables are perfectly measured. To keep the presentation simple I also ignore this distinction in the case of the exogenous variables for the equations of both models. In the diagrams and when I apply these models to data, however, I relax this assumption without loss of generality and present the measurement equations that link the indicators to the latent exogenous variables.

Both the mutual influence and family factor models can be expressed in terms of the same set of reduced-form equations in which the two endogenous variables (the y's) are written as a function of the exogenous variables (the x's). The equations for the Thomson-Williams data are given in 1a and 1b, and the model is diagrammed in Figure 1 of Appendix A. Call this model TW1 (for the first model of the Thomson-Williams data). The b's in equations (1a) and (1b) are reduced-form coefficients. Representing the total effect of each exogenous variable, they combine the direct effects in the structural form of the two models (Duncan, 1975:152–159): the effect of the wives' and husbands' taste for children on their respective plans and the reciprocal effects of each spouse's plans on the other's in the case of the mutual influence model; the effects of the wives' and husbands' taste on the couple factor (the couple's plans) and loadings of plans on the family factor in the case of the family factor model.

The disturbance terms (the u*'s) in 1a and 1b combine the disturbance terms in the structural forms of the mutual influence and family factor models. There are two types of disturbances in the structural forms: errors in equations, which I represent with the letter "u", and errors in variables, which I represent with the letter "e".[3]

(1a) $y_3 = b_{32}x_2 + b_{31}x_1 + u^*_3$

(1b) $y_4 = b_{42}x_2 + b_{41}x_1 + u^*_4$

In the discussion of the models below I refer to b_{31} and b_{42} as "within-person effects" and to b_{32} and b_{41} as "cross-person effects." I refer to b_{31} and b_{42} as within-person effects because they represent the effect of each spouse's taste for children on his or her plans; I refer to b_{32} and b_{41} as cross-person effects because they represent the total effect of one spouse's taste for children on the other spouse's plans.

The Mutual Influence Model

Equations 2a and 2b specify the structure of the mutual influence model, and Figure 2 in Appendix A diagrams this structure. In the mutual influence model of the Thomson and Williams data, I treat the wives' and husbands' respective plans for another child as separate latent variables. Figure 2 diagrams this specification. Although the error variance of a single-indicator measure is usually not identified, I achieve identification by setting the covariance between the disturbances equal to zero and by constraining the variances of the disturbances for each parent's plans to equal one another. In order to focus on the structure of the mutual influence model, however, I shall assume in this section that y_3 and y_4 are perfectly measured.[4]

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3 Errors in equations occur because the model omits from each equation a variety of causes of the dependent variable—for example, the wives' work status, family finances, the number of children already in the family, etc. Errors in variables represent the ubiquitous presence of measurement error which spans the gamut from relatively mechanical problems (respondent error, interviewer error, processing error) to the more fundamental methodological/conceptual problems involved in bridging the gap between a concept and its measure(s).

4 Random error in the measures of the endogenous variables does not bias the estimates of the effects of either the exogenous or endogenous variables in models with reciprocal causation, but the estimate of the covariance between the errors in equations will be downwardly biased (Gillespie and Fox, 1980).

(2a) $y_3 = \beta_{34}y_4 + \Gamma_{31}x_1 + u_3$

(2b) $y_4 = \beta_{43}y_3 + \Gamma_{42}x_2 + u_4$

Equations 3a and 3b spell out the relation between the reduced-form coefficients and disturbances in equations 1a and 1b and the structural coefficients of the mutual influence model in equations 2a and 2b. For example, b_{31} in equation 1a equals $\frac{\Gamma_{31}}{1 - \beta_{34}\beta_{43}}$ in equation 3a, and u^*_4 in equation 1b equals $\frac{\beta_{43}u_3 + u_4}{1 - \beta_{34}\beta_{43}}$ in equation 3b.

(3a) $y_3 = \frac{\beta_{34}\Gamma_{42}}{1 - \beta_{34}\beta_{43}}x_2 + \frac{\Gamma_{31}}{1 - \beta_{34}\beta_{43}}x_1 + \frac{u_3 + \beta_{34}u_4}{1 - \beta_{34}\beta_{43}}$

(3b) $y_4 = \frac{\Gamma_{42}}{1 - \beta_{34}\beta_{43}}x_2 + \frac{\beta_{43}\Gamma_{31}}{1 - \beta_{34}\beta_{43}}x_1 + \frac{\beta_{43}u_3 + u_4}{1 - \beta_{34}\beta_{43}}$

Equations 4a − 4d present the indirect least squares estimates of the the Γ's and β's in equations 2a and 2b in terms of the b's in equations 1a and 1b (Duncan, 1975:71–73).

(4a) $\beta_{34} = \frac{b_{32}}{b_{42}}$

(4b) $\beta_{43} = \frac{b_{41}}{b_{31}}$

(4c) $\Gamma_{31} = b_{31}(1 - \beta_{34}\beta_{43}) = b_{31}(1 - \frac{b_{41}b_{32}}{b_{31}b_{42}})$

(4d) $\Gamma_{42} = b_{42}(1 - \beta_{34}\beta_{43}) = b_{42}(1 - \frac{b_{41}b_{32}}{b_{31}b_{42}})$

If we assume that both β_{34} and β_{43} are less than one, then an important implication of the mutual influence model is that the cross-member (total) effect of an exogenous variable will be less than its within-member (total) effect. (See equations 4a and 4b.) Equations 5a and 5b formally present this result

(5a) If $\beta_{34} < 1$, then $b_{32} < b_{42}$

(5b) If $\beta_{43} < 1$, then $b_{41} < b_{31}$

The assumption that the value of the two reciprocal effects is less than one seems fairly reasonable in the case of family data. First, meaningful interpretation of the coefficients of all models with reciprocal causation requires that the estimates of the reciprocal effects meet a "stability condition" (Joreskog and Sorbom, 1988: 34–36). In the case of two endogenous variables, the stability condition is that the product of the reciprocal effects be less than one.[5] When the reciprocal effects are approximately equal, satisfaction of the stability condition implies that both reciprocal effects will be less than one. Although various combinations of reciprocal effects can have a product of less than one when the coefficients differ, both effects usually will be less than one in the case of family data. The reason stems from the fact that in family data both endogenous variables are typically measured on the same scale. In this case, the variance of the endogenous variables will be approximately the same, so that the unstandardized reciprocal effects will approximately equal the standardized ones. Since standardized slopes are generally smaller than one, the unstandardized reciprocal effects in family data will be less than one even when they differ.

The Family Factor Model

Equations 6a – 6c specify the structure of the family factor model, and Figure 3 in Appendix A diagrams this structure. Adopting Bentler's (1989) notation, I use F_5 to denote the family factor ("F" for factor). Equations 6a and 6b link y_3 and y_4 to the family factor, while equation 6c links the family factor to the two exogenous variables. The important feature of the family factor model is the shift in status of the wives' and husbands' plans from conceptually distinct variables to measures of the family factor F_5. Accordingly, the u's in equations 2a and 2b become e's in

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5 The requirement that the product of the reciprocal effects be less than one stems from the meaning of direct and indirect effects in models with reciprocal causation. When there is reciprocal causation, the direct effect represents the "instantaneous" change in the dependent variable in response to a change in the exogenous variable. The indirect effect represents the sum of an infinite set of successively smaller changes that are produced by the feedback loop of reciprocal effects. In order for this sum to converge (i.e., for the set changes in the dependent variable to be successively smaller), the product of the reciprocal effects must be less than one. (The statement of the stability condition becomes more complex when the number of endogenous variables is greater than two.)

equations 6a and 6b. The only disturbance in the model, u_5, represents the causes of the family factor omitted from the model.

(6a) $$y_3 = \lambda_{35}F_5 + e_3 \qquad (\lambda_{35} = 1)$$

(6b) $$y_4 = \lambda_{45}F_5 + e_4$$

(6c) $$F_5 = \Gamma_{52}x_2 + \Gamma_{51}x_1 + u_5$$

In order to give the F_5 a metric, I set λ_{35} equal to unity (Duncan, 1975:131). I make this constraint explicit in equation 6a, but hereafter it will be implicit. In the analysis of the Thomson and Williams data in the next section, I also set λ_{45} equal to unity because both y_3 and y_4 are measured on the same scale. In this section, however, I let λ_{45} differ from unity in equation 6b to make the results more general.[6]

Equations 7a and 7b specify the reduced-form of the family factor model. The coefficients in these equations correspond to the coefficients in equations 1a and 1b. (For example, b_{41} of equation 1b equals $\lambda_{45}\Gamma_{51}$ in equation 7b, and u^*_4 of equation 1b equals $\lambda_{45}u_5 + e_4$ in equation 7b.) The reduced-form of the family model is much less complex than the reduced-form of the mutual influence model. Note that the structural coefficients in equation (6a) equal the reduced-form coefficients in equation (7a) because $\lambda_{35} = 1$.

(7a) $$y_3 = \Gamma_{52}x_2 + \Gamma_{51}x_1 + u_5 + e_3$$

(7b) $$y_4 = \lambda_{45}\Gamma_{52}x_2 + \lambda_{45}\Gamma_{51}x_1 + \lambda_{45}u_5 + e_4$$

Equations 7a and 7b show that we can use either the ratio of b_{41} to b_{31} or the ratio of b_{42} to b_{32} to estimate λ_{45}. I present this result in equation 8. This result shows that the ratio of the within-person to cross-person effect of one exogenous variable equals the reciprocal of the ratio of the within-person to cross-person effect of the other exogenous variable.

(8) $$\frac{b_{41}}{b_{31}} = \frac{b_{42}}{b_{32}} = \lambda_{45}$$

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6 When the same measurement scale is used, the coefficients will differ only when the validity of the two measures differs.

The Consequences of Applying One Model to Data Generated by the Other

In this subsection I examine the implications of the two models for the reduced-form coefficients (the b's) under three different sets of equality constraints. First, I assume a fully constrained model with equal Γ's and equal β's in the case of the mutual influence model, and equal Γ's and both λ's set equal to unity in the case of the family factor model. Equations 9a and 9b present the respective implications of the two models for the reduced-form coefficients, given these constraints.

(9a) $b_{41} = b_{32} < b_{31} = b_{42}$ (Mutual Influence Model)

(9b) $b_{41} = b_{32} = b_{31} = b_{42}$ (Family Factor Model)

Second, I relax the assumption of equal Γ's but retain the assumption of equal β's in the mutual influence model and the comparable assumption of both λ's set equal to unity in the family factor model.

(10a) $\frac{b_{41}}{b_{31}} = \frac{b_{32}}{b_{42}} < 1$ (Mutual Influence Model)

(10b) $b_{41} = b_{31} \neq b_{32} = b_{42}$ (Family Factor Model)

Third, I relax the assumption of equal β's in the mutual influence model and the comparable assumption of both λ's equal to one in the family factor models, but I retain the assumption that the reciprocal effects are both less than unity in the case of the mutual influence model. (Note that 11b just repeats equation 8.)

(11a) $\frac{b_{41}}{b_{31}}, \frac{b_{32}}{b_{42}} < 1$ (Mutual Influence Model)

(11b) $\frac{b_{41}}{b_{31}} = \frac{b_{42}}{b_{32}} = \lambda_{45}$ (Family Factor Model)

In each set of equations the family factor model is more restrictive than the mutual influence model, but the less restrictive constraints of the mutual influence model are contraindicated by the family factor model. Each equation for the family factor model constrains the effects of a given exogenous variable on the two endogenous variables either to equal one another or, in the

case of equation 11b, to be proportionately equal such that the ratio of the within-person to cross-person effects of one exogenous variable will equal the inverse of this ratio for the other exogenous variable. In contrast, the mutual influence model places no such equality constraints on the within-person and cross-person effects but, instead, implies that cross-person effect will always be less than the within-person effect, a constraint that is contraindicated by the family factor model.

Because it is both more restrictive and specifies the "wrong" restrictions, the family factor model will not fit data generated by a mutual influence process. Fitting an appropriate mutual influence model with the "correct" but less restrictive constraints will significantly improve the fit, and typically the fit will be acceptable on statistical grounds.

The consequences of fitting a mutual influence model to data generated by a family factor structure are both more subtle and more severe. Although the mutual influence model could fit as well or even better than the family factor model, the researcher will have trouble obtaining a solution, and the results he or she obtains will be degenerate. These problems occur because the data do not conform to the stability condition of the mutual influence model. As equation 12 shows, the family factor model implies that the product of the reciprocal effects will equal one rather than be less than one.

$$(12) \qquad \beta_{34}\beta_{43} = \frac{b_{32}b_{41}}{b_{42}b_{31}} = \lambda_{45}\frac{1}{\lambda_{45}} = 1$$

This particular violation of the stability condition poses a logical problem for the mutual influence model since it implies that the reduced-form coefficients for the model have zero as their denominator. (See equations 3a and 3b.) When the family factor model fits the data exactly, the mutual influence model will be undefined, and the researcher will be unable to obtain a solution. In the usual case of approximate fit, however, the researcher may achieve a solution, albeit with difficulty, but will obtain "strange" results. First, β_{34} will approximately equal the reciprocal of β_{43}. When both y_3 and y_4 have the same loading on F_5, therefore, both β's will be close to unity. Second, the estimates of the Γ's will be close to zero. Third, the estimates of the variances for the disturbances—$\sigma_{u_3u_3}$ and $\sigma_{u_4u_4}$—also will be close to zero when the endogenous variables y_3 and

y_4 have been purged of random measurement error by setting the covariance between the disturbances equal to zero. I develop these results below.

The first result simply restates the violation of the stability condition in equation 12. The special case of this violation, β's approximately equal to one when y_3 and y_4 have the same loading on F_5, follows from equation 11b and equations 4a and 4b. The result of near-zero Γ's is implied by equations 4c and 4d, which express the Γ's as a function of the reduced-form coefficients. The calculation of both Γ's involves the multiplication of the reduced-form coefficients b_{31} and b_{42} by the near-zero value $(1 - \beta_{34}\beta_{43})$. The same reasoning applies to the calculation of the variances of the errors in equations when y_3 and y_4 have been purged of random error. This result is shown in equations 13a and 13b. Developed in Appendix B, these equations express the variance of the disturbances as a function of the reciprocal effects and the variances of the reduced-form disturbances, $\sigma_{u^*_3u^*_3}$ and $\sigma_{u^*_4u^*_4}$. Since the numerator of both equations is multiplied by the term $(1 - \beta_{34}\beta_{43})$, the estimates of the disturbance variances, $\sigma_{u_3u_3}$ and $\sigma_{u_4u_4}$ will approach zero when a mutual influence model is applied to data generated by a family factor structure. Note that the generality of this results is limited by my assumption that the covariance between the disturbances of the structural equations, $\sigma_{u_4u_3}$, equals zero. I make this assumption in applying the mutual influence model to both the Thomson-Williams and Jennings-Niemi data in order to estimate the random error in the measures of the endogenous variables.[7]

$$\text{(13a)} \qquad \sigma_{u_3u_3} = (1 - \beta_{34}\beta_{43}) \frac{\sigma_{u^*_3u^*_3} - \beta^2_{34}\sigma_{u^*_4u^*_4}}{1 + \beta_{34}\beta_{43}}$$

$$\text{(13b)} \qquad \sigma_{u_4u_4} = (1 - \beta_{34}\beta_{43}) \frac{\beta^2_{43}\sigma_{u^*_3u^*_3} - \sigma_{u^*_4u^*_4}}{1 + \beta_{34}\beta_{43}}$$

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[7] When this assumption is not made, the estimate of $\sigma_{u_4u_3}$ will be negatively signed. In this case, it is possible to obtain estimates of $\sigma_{u_3u_3}$ and $\sigma_{u_4u_4}$ that are greater than zero. Although I do not develop this result formally, it is consistent with the argument made by Gillespie and Fox (1980) that negatively correlated disturbances in structural equation models with positive reciprocal effects is a symptom of model misspecification.

APPLICATION OF THE TWO APPROACHES TO TWO DATA SETS

The Thomson-Williams Couple Data

Thomson and Williams (1982) present data on 340 white, married couples surveyed in Seattle in 1977. They note that "each of the wives had borne her first or second child during 1973–1976 and was between the ages of 26 and 36 at the time of the survey." Wives and husbands "simultaneously and independently" provided data on their "utility" or "taste" for another child and on their plans for another child. Thomson and Williams use two items per respondent to measure each spouse's taste for children: the extent to which another child would contribute to "a fulfilled family life" (WU1 and HU1 for wife and husband, respectively) and the value of "watching another child grow and develop" (WU2 and HU2). They use only one measure of plans per couple: each member's "estimate of the likelihood that the couple will have another child within the next twenty months" (WP and HP). The correlations, standard deviations, and means for these variables are given in Table 1.[8]

Table 1
Correlations, Standard Deviations, and Means
for 340 Wife-Husband Dyads

	Variables (defined in the Text)					
	WU1	WU2	HU1	HU2	WP	HP
WU1	1.000					
WU2	.470	1.000				
HU1	.460	.270	1.000			
HU2	.312	.223	.495	1.000		
WP	.628	.421	.498	.381	1.000	
HP	.596	.347	.586	.422	.816	1.000
SD's	11.45	10.89	10.73	10.30	2.66	2.60
Means	11.36	22.34	9.75	18.50	3.64	3.66

The measurement portion of the models is presented in equations 14a – 14f. To distinguish measures from latent variables, I use letters to subscript the measures, and I use numbers to sub-

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[8] To make the estimation go faster, I divided the standard deviations by ten.

script the latent variables. The letters x_a through x_d refer to WU1, WU2, HU1, and HU2, while y_w and y_h refer to wife's and husband's respective plans for an additional child.

(14a) WU1 $x_a = x_1 + e_a$

(14b) WU2 $x_b = \lambda_{b1} x_1 + e_b$

(14c) HU1 $x_c = x_2 + e_c$

(14d) HU2 $x_d = \lambda_{d2} x_2 + e_d$

(14e) WP $y_e = y_3 + e_e$

(14f) HP $y_f = y_4 + e_f$

Equations 14a through 14d link the measures of taste for children to their respective latent variables: wife's taste for children (x_1) and husband's taste for children (x_2). The final model discussed by Thomson and Williams (1982), imposes the following constraints on the measures: $\lambda_{b1} = \lambda_{d2}$; $\sigma_{e_a e_a} = \sigma_{e_c e_c}$; $\sigma_{e_b e_b} = \sigma_{e_d e_d}$; and $\sigma_{11} = \sigma_{22}$. I maintain these constraints in the models I describe, although I add others.

Equations 14e and 14f decompose the error in the endogenous variables in the mutual influence model into errors in equations and errors in variables (measurement error). As indicated in the section on the mutual influence model, the error variance of single-indicator measures is typically unidentified. I achieve identification by setting the covariance between the disturbances, $\sigma_{u_4 u_3}$, equal to zero and by setting the variances for the errors in the wives' and husbands' equations, $\sigma_{u_3 u_3}$ and $\sigma_{u_4 u_4}$, equal to one another. (I also set the variances of the measurement errors, $\sigma_{e_e e_e}$ and $\sigma_{e_f e_f}$ equal to one another, but this specification is not needed to identify the model.) I develop this result in Appendix C.

Table 2 contains the goodness of fit statistics for five models of the Thomson-Williams data. The statistics are: the maximum likelihood chi-square (L^2), the degrees of freedom (df), and the probability of obtaining the observed value of chi-square when the model being tested is the correct one (prob.). I obtained these results by using EQS-PC (Bentler, 1989) to analyze the

covariance matrix obtained from the table of correlations and standard deviations in Table 1.

Table 2
Goodness of Fit Statistics for Selected
Models of the Thomson-Williams (1982) Data

Model	Description	L^2	df	prob.
TW1	Reduced-form of Mutual Influence and Family Factor Models	9.088	9	.429
TW2	Thomson & Williams' (1982) Family Factor Model 2D	24.826	10	.006
TW3	Thomson & Williams' Model 2D Plus Constraints	27.462	13	.011
TW4	Mutual Influence Model Fully Constrained	11.838	12	.459
TW5	Family Factor Model Plus Correlated Disturbances	11.060	11	.438

The first model in Table 2 (TW1) is the reduced-form of the mutual influence and family factor models. (See equations 1a and 1b and Figure 1, Appendix A.) This model imposes no constraints on the relations between the (latent) exogenous variables and the wives' and husbands' respective plans for another child. It also aggregates the errors in variables for the endogenous variables with the errors in equations. Consequently, the goodness-of-fit statistics test just the validity of the constraints Thomson and Williams impose on the relations between the measures of the exogenous variables, the wives' and husbands' respective taste for children. The results support these specifications.[9]

The second model (TW2) is Thomson and Williams' (1982:1004) family factor model 2D. (See equations 6a − 6c and Figure 3, Appendix A.) The only additional constraint it imposes on the data, vis a vis model TW1, results from the specification of the family factor F_5, the couple's plans for another child. (See equation 11b.) The third model (TW3) is a family factor model with

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[9] In all models, the estimates of the variances and covariances for the exogenous variables and their measures remain the same: $\lambda_{b1} = \lambda_{d2} = \lambda = .653$; $\sigma_{e_a e_a} = \sigma_{e_c e_c} = .363$; $\sigma_{e_b e_b} = \sigma_{e_d e_d} = .754$; $\sigma_{11} = \sigma_{22} = .868$.

equality constraints I imposed on the Γ's, the λ's, and measurement error variances.[10] (See equation 9b.) Neither TW2 nor TW3 provide an acceptable fit to the data, but adding the new constraints to model TW3 does not significantly reduce its fit.

Model TW4 is a fully constrained mutual influence model that is comparable to the family factor model TW3. It differs from TW3 only in that the β's have to be estimated, while both λ's in TW3 are fixed at one. (Compare equations 9a and 9b.) The estimation of the β's costs a degree of freedom. This cost is warranted, however, by the increase in the fit of the model to the data to one that is acceptable statistically. Model TW5 also fits the data well. Diagrammed in Figure 4, it represents my modification of a model presented by Thomson and Williams (1984) in their later reanalysis of the data. They retain their family factor model but improve its fit by allowing certain measurement error variances to covary. I describe this model in more detail below after comparing model TW3 with model TW4.

Table 3 contains the unconstrained reduced-form coefficients for model TW1 and the constrained reduced-form coefficients for the family factor and mutual influence models, TW3 and TW4, respectively. The table also contains the formulas for the reduced-form coefficients for models TW3 and TW4. Note the equalities in the reduced-form coefficients for models TW3 and TW4 that result from the equality constraints imposed on the structural coefficients of the respective models. (See equations 9a and 9b.)

As Table 3 shows, the reduced-form coefficients for the mutual influence model (TW4) agree better than the reduced-form coefficients for the family factor model (TW3) with the unconstrained b's. In the case of both the unconstrained b's and the b's implied by the mutual influence model TW4, the cross-person effects are smaller than the within-person effects. This comparison shows why the mutual influence model fits the data better than either of the family factor models (TW2 or TW3). Whereas the family factor model implies that $(b_{31}/b_{41}) = b_{32}/b_{42})$, the first ratio is almost three times greater than the second one: (1.66/1.10 = 1.51; 1.27/0.66 = 0.52).

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[10] Specifically, I set $\Gamma_{51} = \Gamma_{52} = \Gamma$, $\lambda_{45} = 1$, and $\sigma_{u_3u_3} = \sigma_{u_4u_4} = \sigma_{uu}$.

Table 3

Reduced-Form Coefficients[a] for Models TW1, TW3, an l TW4 of the Thomson-Williams Data

Reduced-form (TW1)		Family Factor (TW3)		Mutual Influence (TW4)	
Symbol	Values	Symbol	Values	Symbol[b]	Values
b_{31}	1.66	Γ_{51}	1.18	$\Gamma_{31}C$	1.484
b_{32}	.66	Γ_{52}	1.18	$\beta_{34}\Gamma_{42}C$	.885
b_{41}	1.10	Γ_{51}	1.18	$\beta_{43}\Gamma_{31}C$	.885
b_{42}	1.27	Γ_{52}	1.18	$\Gamma_{42})C$	1.484

[a] The set of coefficients in the symbol column for model TW1 denote the unconstrained reduced-form coefficients. For models TW3 and TW4, the numbers in the values column equal the constrained reduced-form coefficient implied by the model.

[b] $C = \frac{1}{1 - \beta_{34}\beta_{43}}$.

Table 4 contains the structural coefficients for the family factor and mutual influence models, TW3 and TW4, respectively.[11] The coefficients in both models are statistically significant at the .01 level. The results for the mutual influence model show that wives' and husbands' respective plans for another child are equally affected by their respective utility for children (.957) and by their spouse's plans (.596). Because of this parsimony, I regard model TW4 as a useful template for mutual influence models of multiple respondent data. Note, however, that the reduced-form coefficients for model TW1 in Table 3 suggest that wives' plans affect husbands' plans more than husbands' plans affect wives' plans. Although this difference does not achieve conventional levels of statistical significance, the power of this test is low (Matsueda and Bielby, 1986). Researchers may wish to relax this equality constraint, therefore, if there are theoretical or other empirical reasons to do so.

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[11] Note that since λ_{45} equals unity, the reduced-form coefficients equal the structural-form coefficients the Γ's in the case of model TW3.

Table 4
Structural Coefficients for Models TW3
and TW4 of the Thomson-Williams Data

Family Factor (TW3)		Mutual Influence (TW4)	
Symbol	Values	Symbol	Values
$\Gamma_{52} = \Gamma_{51}$	1.18	$\Gamma_{42} = \Gamma_{31}$	.957
$\lambda_{45} = \lambda_{35}$	1.00	$\beta_{34} = \beta_{43}$	.596

Other results for model TW4 not reported in the Table 4 are the variances in the disturbances of plans for children and the variances in the measurement errors in reporting plans. Both quantities were set to be equal across husbands and wives ($\sigma_{u_3u_3} = \sigma_{u_4u_4} = .596$;[12] $\sigma_{e_ee_e} = \sigma_{e_fe_f} = .928$). These two sets of equalities means that the measures of the latent variables are equally reliable and valid for husbands and wives. This feature increases the parsimony of TW4 and, therefore, adds to its attractiveness as a template for the mutual influence model.

I separated the disturbances in plans from the disturbances in the reports of plans mainly to provide another test of the mutual influence model. As argued in the previous section, applying a mutual influence model to data generated by a family factor structure yields a near-zero variance of the errors in equations. The fact that the Thomson-Williams data yield non-zero estimates of variances in both errors in equations and errors in variables provides additional evidence that the mutual influence model does a better job than the family factor model at representing the Thomson-Williams' data. Another reason for separating errors in equations and errors in variables is that failure to do this results in a covariance between the disturbances of -1.186, which translates into a correlation of $-.597$. Gillespie and Fox (1980) show that the disturbances of positively related variables will correlate negatively as an artifact of different kinds of misspecification in models with reciprocal causation. A relatively benign misspecification is the as-

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[12] The fact that the reciprocal effects equal the variances of the disturbances is coincidental.

sumption that the endogenous variables are perfectly measured. Correcting for this problem in the Thomson-Williams data by setting $\sigma_{u_4u_3}$ equal to zero yields reasonable estimates of the reliability of the husbands' and wives' plans of approximately .87.[13]

Although Thomson and Williams (1982:1006) acknowledge that their analysis does not exhaust the possible models for their data, they are not particularly troubled by the lack of fit of their family factor model 2D (TW2) to the data. They claim that the power of the maximum-likelihood statistic makes the fit of their model is reasonable. In a later article, they modify model 2D by allowing the errors for the measures of taste and plans for another child to covary within members. They further refine the model by constraining the comparable covariances across husbands and wives to equal one another (Thomson and Williams, 1984).[14] I further modify their model by specifying equal Γ's. The result is the family factor model TW5 in Table 2. Their motivation for this new model is the possible bias in the estimates of the Γ's that results from the failure to include these covariances rather than by the misfit of their 1982 model. As the goodness-of-fit statistics show, however, adding these covariances increases the fit to a level that statistically is both acceptable and a significant improvement over the fit of their 1982 model (while producing only marginal changes in the estimates of the Γ's).

Thomson and Williams (1982:633–634) claim that these covariances are needed because of the "methods effects" specific to each member of the couple in their response to questions. From the perspective of the mutual influence model, however, these covariances simply account for the excess within-member covariance between taste for children and plans that is not accounted for by

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13 The formulas for calculating the respective reliabilities of husbands' and wives plans are: $1 - \frac{\sigma_{e_e e_e}}{\sigma_{y_e y_e}}$ and $1 - \frac{\sigma_{e_f e_f}}{\sigma_{y_f y_f}}$. The respective reliabilities are .863 and .867. Estimating the random error variances by setting $\sigma_{u_4u_3}$ equal to zero is admittedly arbitrary; the concerned reader is invited to conduct sensitivity analyses by experimenting with other values of either $\sigma_{u_4u_3}$ or $\sigma_{e_e e_e}$ and $\sigma_{e_f e_f}$.

14 Specifically, they let $\sigma_{e_e e_a} = \sigma_{e_f e_c}$ and $\sigma_{e_e e_b} = \sigma_{e_f e_d}$.

the family factor model. Although we cannot unequivocally rule out their family factor model TW5, the mutual influence model TW4 seems more parsimonious. Model TW5 assumes the existence of four unobserved variables. Each is independent of the others. Each affects a specific measure of the husbands' or wives' taste in children. The two that affect the measures of wives' taste both affect the wives' plans, while the other two that affect husbands' taste both affect the husbands' plans. In contrast, the mutual influence model TW4 achieves an acceptable fit simply by allowing the wives' and husbands' respective plans to influence one another.

The Jennings-Niemi "Triads" Data

The data for the second set of analyses come mainly from a 1965 survey of a national sample of high school seniors and their parents (ICPSR, 1971). Directed by M. Kent Jennings, the survey focused on the political attitudes and behavior of the students and their parents. In 1973, Jennings and his associate Richard G. Niemi directed a second wave of interviews with the "students" (now around 25 years old) and their parents. I supplement the 1965 data with the students' 1973 reports of their schooling since high school (ICPSR, 1981).

The sampling plan for the parent (or parent substitute) interviews in 1965 divided the families into three random subsamples of approximately equal size. Only one parent was interviewed in the first two subsamples; both parents were interviewed in the third.[15] Even though complete information from the student and both parents is available in the last subsample, I use the data from all three subsamples in the analysis. I restrict the analysis, however, to white families, to families in which the student reported in 1965 that he or she was living with both parents, and to families in which at least one parent was interviewed.[16] This restriction plus the

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15 Only one of the parents in the triad sample was reinterviewed in the 1973 wave. The University of Michigan Inter-university Consortium for Political and Social Research has just made available the data from a third wave of students and parents conducted in 1981.

16 Restricting the sample to students living with both father and mother eliminates both single-parent families and families in which one or both of the parents is a step-parent. I eliminate these cases, as well as black families, from the analysis because of the possibility of substantial differences between the structure that underlies the data in these cases and the structure that underlies the data in the case of white, intact families. Needless to say, this possibility should be investigated in future research.

elimination of data through panel attrition and the listwise deletion of cases missing data on one or more variables (other than those excluded by virtue of the sampling plan for parents) yielded a sample of 233 student-mother-father triads, a sample of 269 student-mother dyads, and a sample of 258 student-father dyads.[17]

I use LISREL7 (Joreskog and Sorbom, 1988) to analyze the data. In order to use the dyads in the analysis, I adopt recently developed techniques for dealing with incomplete data in structural equation models (Allison, 1987; Bentler, 1989; Joreskog and Sorbom, 1988:259–261). I discuss this and other aspects of the analysis in Appendix D. In addition, Appendix D describes results that are not central to the discussion below.

Table 5 contains a list of the ten observed variables I use in the analysis. Six of the ten variables are measures of the two latent exogenous variables: father's schooling and mother's schooling (x_1 and x_2, respectively).. Each member of the family provided a report of these variables. There are four observed endogenous variables: each family member's plans for the post-secondary schooling of the student obtained in 1965 and the student's schooling reported in 1973. I treat the reported plans as fallible measures of a latent variable.[18] I treat the student's report of his or her schooling as measured without error.[19] All ten variables are measured in terms of the years of school completed. Assigning years of schooling to these variables involved extensive recoding; Appendix D reports the details of these recodes.

Table 6 contains the mean and standard deviation of each variable and the zero-order correlations between each pair of variables for the 233 student-mother-father triads. The results for

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17 A few of the dyads were intended to be triads according to the sample, but only one parent was available for interview. In a few other dyads the parent interviewed was a substitute for the one designated by the sampling plan.

18 In the mutual influence model, I use y_g, y_h, and y_i, respectively to distinguish the observed values of father's, mother's, and student's plans from the latent variables y_3, y_4, and y_5.

19 This treatment aggregates the measurement error and the errors in the equation for the student's 1973 schooling. Because this variable is always a dependent variable, this aggregation does not bias the estimates of the effects of the other variables on the student's schooling.

Table 5
Selected Variables From the Jennings-Niemi Data (1965, 1973)

Variable	Mnemonic	Description
y_6	SED	Student's Schooling (1973)
y_5	SPLN	Student's Plans for Student's School (1965)
y_4	MPLN	Mother's Plans for Student's School (1965)
y_3	FPLN	Father's Plans for Student's School (1965)
x_a	SFED	Student's Report of Father's Education (1965)
x_b	MFED	Mother's Report of Father's Education (1965)
x_c	FFED	Father's Report of Father's Education (1965)
x_d	SMED	Student's Report of Mother's Education (1965)
x_e	MMED	Mother's Report of Mother's Education (1965)
x_f	FMED	Father's Report of Mother's Education (1965)

the student-mother and student-father dyads are given in Appendix D. Looking at the correlations in Table 6, we see extremely high "agreement" between all three members' reports of the mother's and father's schooling. This agreement attests to the high quality of these reports as measures of the parent's schooling. We also see high agreement between each member's plans for the student's schooling. Finally, notice that all three members' plans correlate equally well (and highly) with the student's schooling (reported eight years later).

The measurement model for the exogenous variables is given in equations 15a – 15f. The equations set the loadings of the parents' reports on the true values of mother's and father's schooling equal to one. I set the loadings of the student's reports equal to one another but let them differ from the parents' loadings. In addition, I included the loading of the student's plans on the "true" values of this variable in the equality constraint. (See equations 16c, 17c, and 18e.)[20]

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[20] In the family factor model JN2 ($\lambda_{a1} = \lambda_{d2} = \lambda_{57} = \lambda = .945$); in the mutual influence

Table 6
Correlations, Standard Deviations, and Means
for 233 Student-Mother-Father Triads

	Variables (defined in Table 5)									
	SED	SPLN	MPLN	FPLN	SFED	MFED	FFED	SMED	MMED	FMED
SED	1.00									
SPLN	.641	1.00								
MPLN	.643	.764	1.00							
FPLN	.629	.750	.789	1.00						
SFED	.461	.488	.448	.444	1.00					
MFED	.470	.487	.436	.460	.940	1.00				
FFED	.476	.503	.454	.466	.926	.962	1.00			
SMED	.394	.410	.369	.378	.531	.544	.548	1.00		
MMED	.389	.398	.374	.393	.551	.570	.590	.908	1.00	
FMED	.426	.409	.384	.371	.565	.584	.607	.902	.938	1.00
SD's	1.92	1.52	1.63	1.64	3.13	3.21	3.15	2.38	2.52	2.43
Means	14.5	14.7	14.5	14.4	11.9	12.0	12.1	12.0	12.0	12.0

I also set the error variances of the parents' reports equal to one another ($\sigma_{e_b e_b} = \sigma_{e_c e_c} = \sigma_{e_e e_e} = \sigma_{e_f e_f} = .374$) and set the error variance for the two student reports equal to one another but let them differ from those of the parents ($\sigma_{e_a e_a} = \sigma_{e_d e_d} = .794$).[21] I maintain these constraints in all four models described below, although as I point out in Appendix D, the estimates of some of these error variances differ across the sample of triads and the two samples of dyads.

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model JN3 ($\lambda_{a1} = \lambda_{d2} = \lambda_{i5} = \lambda = .949$); in the couple factor model JN4 ($\lambda_{a1} = \lambda_{d2} = \beta_{57} = \lambda = .946$).

21 The differences between the estimates of the measurement error variances for the parents and students is statistically significant at the .01 level. The fact that student error variances are greater and the loadings are smaller means that the student's reports of his or her parents' schooling are slightly less reliable and valid than the parents' reports.

(15a) SFED $x_a = \lambda_{a1} x_1 + e_a$

(15b) MFED $x_b = x_1 + e_b$

(15c) FFED $x_c = x_1 + e_c$

(15d) SMED $x_d = \lambda_{d2} x_2 + e_d$

(15e) MMED $x_e = x_2 + e_e$

(15f) FMED $x_f = x_2 + e_f$

Table 7 contains the goodness of fit statistics for four models of the Jennings-Niemi data: JN1, JN2, JN3, and JN4. Diagrammed in Figures 5 – 8 of Appendix A, the first three models are comparable to the reduced-form, family factor, and mutual influence models of the Thomson-Williams data, except that they contain two additional endogenous variables.[22] The fourth model is a couple factor model and also is comparable to the family factor model for the Thomson-Williams data.

Table 7
Goodness of Fit Statistics for Selected
Models of the Jennings-Niemi Data

Model	Description	L^2	df[a]	prob.
JN1	Reduced-form Coefficients	103.45	103	.469
JN2	Family Factor Model	113.03	115	.534
JN3	Mutual Influence Model[b]	112.54	113	.494
JN4	Couple Factor Model	113.01	114	.509

[a] The degrees of freedom given in the LISREL output are incorrect and have to calculated by hand. (See Appendix D.)

[b] Test results for the mutual influence model may be incorrect because the model did not converge after 200 iterations.

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22 To avoid cluttering the diagrams, I omit the direct effects of father's and mother's schooling on student's schooling.

Similar to model TW1, the reduced-form model for the Jennings-Niemi data, JN1, provides a basis against which to compare the three other models. It consists of the reduced-form slopes of the four endogenous variables on the true values of the father's and mother's schooling plus the variances and covariances of the reduced-form disturbances for the endogenous variables (the u*'s). Since it places no constraints on the effects of parents' schooling on the endogenous variables or on the relations among the endogenous variables, the goodness-of-fit statistics for the model in Table 7 simply test the constraints imposed by the measurement model in equations 15a – 15f.[23] As these statistics show, the data are consistent with the constraints. An important feature of these data is that in all models the variables appear to be free of systematic measurement error both in the form of correlated measurement error terms and in the form of one measure of a latent variable loading on another latent variable.[24]

The family factor model, JN2, treats the student's and parents' respective 1965 plans for the student's schooling as measures of a family factor (F_7). Equations 16a – 16e specify the structure of this model for the endogenous variables. Equations 16a – 16c specify the effects of the family factor F_7 on the respective plans of the father, mother, and student, $y_3 - y_5$. Equation 16d specifies the effects of father's and mother's schooling on the family factor. Equation 16e specifies the effects of the family factor, father's schooling, and mother's schooling on the student's schooling. In estimating the parameters of equations 16d and 16e, I set the effects of father's and mother's schooling on the endogenous variables equal to one another: ($\Gamma_{71} = \Gamma_{72}$; $\Gamma_{61} = \Gamma_{62}$). I also set the error variances in the father's, mother's, and student's respective plans equal to one another: ($\sigma_{e_3e_3} = \sigma_{e_4e_4} = \sigma_{e_5e_5} = .615$). As the goodness-of-fit statistics show, this model fits the data well.

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23 As stated above, however, the model uses the loading of the student's reports of his or her parents' schooling on the true values of these variables to provide the metric for the student's plans.

24 In all models, the largest modification index pertains to the covariance between the error in the student's and father's reports of father's schooling in the triads subsample. Although freeing this parameter would yield a significant improvement in fit, I do not do this for a variety of reasons. See Appendix D for further discussion.

(16a) $y_3 = F_7 + e_3$

(16b) $y_4 = F_7 + e_4$

(16c) $y_5 = \lambda F_7 + e_5$

(16d) $F_7 = \Gamma_{72}x_2 + \Gamma_{71}x_1 + u_7$

(16e) $y_6 = \beta_{67}F_7 + \Gamma_{62}x_2 + \Gamma_{61}x_1 + u_6$

The mutual influence model, JN3, specifies reciprocal effects of the true values of mother's and father's plans on one another but recursive effects of each parent's plans on (the true values of) the student's plans. Equations 17a − 17g specify the structure of this model for the endogenous variables. Equations 17a − 17c are the measurement equations. As in mutual influence model for the Thomson-Williams data, TW4, the structure of JN3 is sparse enough to permit identification of both the error variances of the single-indicator measures and the disturbance variances of the latent variables they measure. (See Appendix C.) Note that to provide continuity with the family factor model JN2, I set the loading of the observed values of student's plans (y_i) on the true values (y_5) in equation 17c equal to the loadings of the student's reports of mother's and father's schooling on these variables ($\lambda_{i5} = \lambda_{a1} = \lambda_{d2} = \lambda$). Equations 17d and 17e reproduce the mutual influence model in equations 2a and 2b. Equation 17f specifies the effects of mother's and father's plans, y_3 and y_4, on the student's plans, y_5. Finally, equation 17g specifies the effects of the student's plans, father's schooling, and mother's schooling on student's schooling. Note that while father's and mother's schooling have direct effects on the student's schooling, their respective plans affect student's schooling only indirectly via their effect on the student's plans.

(17a) $y_g = y_3 + e_g$

(17b) $y_h = y_4 + e_h$

(17c) $y_i = \lambda y_5 + e_i$

(17d) $y_3 = \beta_{34} y_4 + \Gamma_{31} x_1 + u_3$

(17e) $y_4 = \beta_{43} y_3 + \Gamma_{42} x_2 + u_4$

(17f) $y_5 = \beta_{54} y_4 + \beta_{53} y_3 + u_5$

(17g) $y_6 = \beta_{65} y_5 + \Gamma_{62} x_2 + \Gamma_{61} x_1 + u_6$

The mutual influence model for the Jennings-Niemi data, JN3, contains the comparable equality constraints set in the mutual influence model for the Thomson-Williams data (TW4): equal within-person effects of parents' schooling on parents' plans variables ($\Gamma_{31} = \Gamma_{42}$), equal reciprocal effects ($\beta_{34} = \beta_{43}$), equal effects of parents' schooling on student's schooling ($\Gamma_{61} = \Gamma_{62}$), and equal effects of parents' plans on student's plans ($\beta_{53} = \beta_{54}$). With respect to the error variances, I set both the measurement error variances and the disturbance variances equal to one another: ($\sigma_{e_g e_g} = \sigma_{e_h e_h} = \sigma_{e_i e_i} = .614$; $\sigma_{u_3 u_3} = \sigma_{u_4 u_4} = \sigma_{u_5 u_5} = .002$).

Estimating the (equal) reciprocal effects of father's and mother's plans on one another, rather than setting the loadings of their plans on the family factor equal to one, costs the mutual influence model JN3 one degree of freedom. (Compare equations 9a and 9b.) The second degree of freedom is lost by estimating the (equal) effects of mother's and father's plans on the true values of the student's plans, rather than letting these effects be one as implied by the family factor model. As the goodness-of-fit statistics show, relaxing these constraints does not significantly improve the fit of JN3 over JN2. Moreover, the estimates of the parameters of JN3 are problematic because the estimates failed to converge after 200 iterations.[25]

The final model, JN4, is a couple factor model. It modifies the family factor model JN2 by

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[25] The problem of convergence may lie the application of the procedures for incomplete data since I did not encounter this problem when using just the triads data.

limiting the "family" factor to father's and mother's plans. Equations 18a – 18f specify the structure of this model for the endogenous variables. Like the first family factor model JN2, father's and mother's schooling equally affect the couple factor F_7 and the student's schooling y_6. Unlike the first three models, I specified a unit loading of the student's report of his or her plans (y_i) on the true values of this variable (y_5). I maintain the continuity of JN4 with JN2, however, by setting the effect of the couple factor on the true values of the student's plans equal to the loadings of the student's report of father's and mother's schooling on the true values of these variables ($\beta_{57} = \lambda_{a1} = \lambda_{d2} = \lambda$). With this specification, the only difference between the family factor model JN2 and the couple factor model JN4 is that the latter adds an errors in equations variance for the student's plans ($\sigma_{u_5u_5}$) to the model. The result of this modification is virtually no increase in the fit of the model and a negative estimate of the disturbance variance ($\sigma_{u_5u_5} = -.008$). Although the couple factor model is theoretically an attractive alternative to the family factor model, the data do not support it, so I do not discuss it further.

(18a) $y_3 = F_7 + e_3$

(18b) $y_4 = F_7 + e_4$

(18c) $y_i = y_5 + e_i$

(18d) $F_7 = \Gamma_{72}x_2 + \Gamma_{71}x_1 + u_7$

(18e) $y_5 = \beta_{57}F_7 + u_5$

(18f) $y_6 = \beta_{65}y_5 + \Gamma_{61}x_1 + \Gamma_{62}x_2 + u_6$

The results in Table 8 provide more information on the relative merits of family factor model JN2 over the mutual influence model JN3. Comparable to Table 3 for the Thomson-Williams data, Table 8 contains the unconstrained reduced-form coefficients for the reduced-form model JN1 and the constrained reduced-form coefficients for models JN2 and JN3. Looking at the unconstrained reduced-form coefficients, we see that although the reduced-form coefficients for mother's schooling are somewhat smaller than the comparable slopes for father's schooling, the most noteworthy feature is their overall similarity. In particular, the within-member and cross-

member slopes do not differ appreciably even in the case of the mutual influence model which implies a difference. Consequently, the proportional equality implied by the family factor model fares much better with the Jennings-Niemi data than it does in the case of the Thomson-Williams data.

Table 8

Reduced-Form Coefficients[a] for Models
JN1 – JN4 of the Jennings-Niemi Data

Reduced-form (JN1)		Family Factor (JN2)		Mutual Influence (JN3)	
Symbols	Values	Symbol	Values	Symbol[b]	Values
b_{31}	.159	Γ_{71}	.133	$\Gamma_{31}C$	.127
b_{32}	.102	Γ_{72}	.133	$\beta_{34}\Gamma_{42}C$	.123
b_{41}	.155	Γ_{71}	.133	$\beta_{43}\Gamma_{31}C$	.123
b_{42}	.089	Γ_{72}	.133	$\Gamma_{42}C$	.127
b_{51}	.161	$\lambda\Gamma_{71}$	.126	$\beta_{53}(b_{31} + b_{32})$	.115
b_{52}	.098	$\lambda\Gamma_{72}$	.126	$\beta_{54}(b_{41} + b_{42})$	.115
b_{61}	.188	$\beta_{65}\Gamma_{71} + \Gamma_{61}$	.180	$\beta_{65}b_{51} + \Gamma_{61}$	.172
b_{62}	.168	$\beta_{65}\Gamma_{72} + \Gamma_{62}$	.180	$\beta_{65}b_{52} + \Gamma_{62}$	.172

[a] The set of coefficients in the symbol column for model JN1 denote the unconstrained reduced-form coefficients. For models JN2 and JN3, the numbers in the values column equal the constrained reduced-form coefficient implied by the model.

[b] $C = \frac{1}{1 - \beta_{34}\beta_{43}}$

Table 9 contains the structural coefficients for the family factor and mutual influence models of the Jennings-Niemi data, JN2 and JN3. Looking at the structural coefficients for the mutual influence model JN2, we see the problems that result from fitting a mutual influence model to data generated by a family factor structure. The estimate of the two reciprocal effects is close to one (.976), so that the stability condition is violated, and the estimate of the Γ's is close to zero

(.006). In addition, the variances for the errors in equations also are close to zero ($\sigma_{u_3u_3} = \sigma_{u_4u_4} = \sigma_{u_5u_5} = .002.$)[26] These results, plus the failure of the mutual influence model to improve significantly on the acceptable fit of the family factor model, plus the failure of the mutual influence model to converge, provide strong evidence for the family factor model JN2 over the mutual influence model JN3.

Table 9
Structural Coefficients for Models JN2
and JN3 of the Jennings-Niemi Data

Family Factor (JN2)		Mutual Influence (JN3)	
Symbol	Values	Symbol	Values
$\Gamma_{72} = \Gamma_{71}$	.133	$\Gamma_{42} = \Gamma_{31}$	.006
$\lambda_{47} = \lambda_{37}$	1.00	$\beta_{34} = \beta_{43}$	.976
λ_{57}	.945	$\beta_{54} = \beta_{53}$	.487
$\Gamma_{62} = \Gamma_{61}$	.072	$\Gamma_{62} = \Gamma_{61}$	.072
β_{67}	.811	β_{65}	.824

Interpreted substantively, the results for family factor model JN2 show that father's and mother's schooling equally affect the family's plans for the students post-secondary schooling (.133) which in turn strongly affects the student's schooling (.811). In addition, father's and mother's schooling also have weak but significant direct effects on the student's schooling (.072), so that 51.7% of the variance in the student's schooling is explained by the father's and mother's schooling and family plans.[27] Because it has the same parsimony as the mutual influence model

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26 In the mutual influence model, the estimates of the reciprocal effects are within one standard deviation of one, and both the estimates of the effects of mother's and father's schooling on their respective plans and the estimates of errors in equations variances are within one standard error of zero.

27 The direct effects of father's and mother's schooling occur possibly because the model omits variables, other than family plans, that intervene between the student's schooling and that

of the Thomson-Williams data TW4, model JN2 provides a useful template for family factor models.

One could argue, of course, that the respondent's schooling could be specified as part of the family factor. Such a respecification would be of no consequence for the ten variables in the Jennings-Niemi data set. Formally, this respecification simply involves relabelling β_{67} as λ_{67} and u_6 as e_6. Instead, a test of this modification requires adding new variables to the model. To the extent that the relations between the student's schooling and these variables differ from the relations between the family factor and these variables, one can justify treating the student's schooling as a distinct variable. The fact that both father's and mother's schooling directly affect the student's schooling provides an example of this support.

DISCUSSION

Both the mutual influence and family factor models attempt to portray the structure that underlies the substantial correlations that often occur between the endogenous variables in family data: the correlation in the Thomson-Williams' data between husband's and wife's plans for another child, and the strong correlations in the Jennings-Niemi data between student's, mother's, and father's plans for the student's schooling. As both the formal discussion and the empirical analyses in this paper show, the key to the validity of the model is not the size of these correlations but the fit to the data of the constraints on the reduced-form regressions of the endogenous variables on the exogenous variables. The mutual influence model implies that the cross-person effects will be smaller than the within-person effects; the family factor model implies that the ratio of the two effects will equal the ratio of the loadings of the endogenous variables on the family factor.

Note that this argument extends beyond family data to any situation in which either reciprocal causation or a latent variable provide a theoretically plausible explanation of the correlation between two variables. An irony of the analyses of the family data, however, is that the

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of his parents: for example, father's occupation, family size, family income, the student's grades, his or her track, etc. We also should not ignore the possibility that these estimates are confounded by genetic variables that affect school performance.

model that works best for each data set seems less plausible substantively than the model that does not work. In the case of the Thomson-Williams data, for example, one would think that the plans to have an additional child surely represent a collective decision of the couple rather than individual decision of the husband and wife. Yet the data do not agree. In the case of the Jennings-Niemi data, on the other hand, the mutual influence model seems more appropriate because the status-attainment literature tends to treat the student's attainment as the result of individual action and decision making.[28] Yet the remarkable feature of the Jennings-Niemi data is that the plans of the student and his or her parents appear to represent a collective understanding or decision regarding the student's post-secondary education, and it is this collective understanding, not the student's individual plans, that intervenes between the parents' schooling and that of the student.

The fact that theoretical and/or methodological beliefs can predispose us toward one model or the other highlights the need to be aware of both models. Otherwise, confronted by data for which the model of choice does not work, the researcher may attempt to fix the wrong model rather than specify the right one. Both Thomson and Williams' 1982 and 1984 papers, for example, underscore the need for researchers enamored with the family factor model to be aware of the possibility that a mutual influence model may better fit the data. Their 1982 article illustrates the problem with the widespread practice of explaining away the poor fit of a model as being due to the power of the maximum-likelihood statistic. Their 1984 article illustrates the problems that can arise from uncritical use of correlated errors to achieve a satisfactorily fitting model. While allowing correlated measurement error is a useful and even necessary strategy in some situations, the researcher has to be careful that he or she not fall in the trap of using this strategy to salvage a preferred model when a different model is called for.

In this same vein, it is interesting to speculate on what a researcher would make of the results for the mutual influence model (JN3) for the Jennings-Niemi data if he or she were

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[28] The work of Hauser and others on the status attainment of siblings stands out as an exception to this trend.

unaware of the possibility of a family factor model. In response to the failure of the estimates to converge, the researcher might conduct a fruitless search for instrumental variables to identify the reciprocal effects. Or the researcher might use the inequality constraints in EQS (Bentler, 1989:64) to force the values of the Γ's and the disturbance variances to be greater than zero and the values of the β's to be less than one. Or, despite the lack of convergence, the researcher might simply present the results as real rather than as an artifact of specifying the wrong model.

Having emphasized the difference between the mutual influence models, I want to conclude by discussing the interpretation of the family factor model as a limiting case of the mutual influence model in which the members have achieved a consensus or near consensus. This interpretation is suggested by Friedkin and Cook's (1990) work on interpersonal influence in small groups. Friedkin and Cook (1990) use the results of an experiment on 50 four-person groups to help specify a dynamic model of interpersonal influence. In their experiment, they first assessed the subjects' "initial opinions" on five issues (y_1). They allowed the subjects 20 minutes to discuss the issue with one or more group members and then assessed the subjects' "settled opinions" on the issues (y_∞). The results of the experiment were a consensus of opinions on all but 40 of the 250 trials, and the subject's settled opinion was close to the group mean of the members' initial opinions.[29]

Equations 19a – 19c present the general model they use.[30] In this model $\mathbf{Y}_1$ is an nx1 vector of individuals' initial opinions, $\mathbf{Y}_t$ is an nx1 vector of individuals' opinions at time t (t = 1, 2, 3, ... , ∞), and $\mathbf{Y}_\infty$ is an nx1 vector of the individuals' settled opinions. The symbol **X** is an nxk matrix of k exogenous influences on the individuals' initial opinions; Γ_0 is a kxk matrix of coefficients that link the exogenous influences to the subjects' initial opinions; β is a scalar that

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29 The subjects were college students. The groups differed by type of communication structure (star, kite, circle, slash, and common), composition by sex (25 groups of men and 25 groups of women), and the order in which the issues were presented. The authors account for differences between the subject's settled opinion and the group mean of initial opinions by the group's communication structure.

30 I alter their notation to make it compatible with the notation used to represent the mutual influence model.

represents the degree of interpersonal influence in the group, and α is a scalar that weights the effects of the exogenous variables at time t = 1, 2, 3, ... ∞. Finally, **W** is an nxn matrix that represents the distribution of interpersonal influence within the group. Assuming that this influence is a finite resource, Friedkin and Cook specify that $w_{ij} < 1$ and that the interpersonal influences on any person sum to one.

(19a) $\mathbf{Y}_1 = \mathbf{X}\Gamma_0$

(19b) $\mathbf{Y}_t = \beta \mathbf{W}\mathbf{Y}_{t-1} + \alpha \mathbf{X}\Gamma$

(19c) $\mathbf{Y}_\infty = [\mathbf{I} - \beta \mathbf{W}]^{-1} \alpha \mathbf{Y}_1$

In their adaptation of the model to describe the results of their experiment, Friedkin and Cook, first, allow β, the degree of interpersonal influence in the group, to increase over time and, second, specify that $\alpha = (1 - \beta)$ in order to allow the influence of the exogenous variables on the individuals' opinions to decrease as the members' interpersonal influence increases.[31] In the limiting condition of consensus among the group members, they specify that the interpersonal effect parameter β approaches unity, so that α approaches 0, and equation 19c approaches equation 20. Equation 20 expresses the subject's settled opinion as a weighted average of the group's initial opinion. In groups where the power is evenly distributed and each member is able to communicate directly with all other members, w_{ij} equals $1/n_j$, where n_j is the number of members in the jth group. In this case the limit of the weight matrix ($\mathbf{W}^\infty$) just equals itself (**W**), and the members' settled opinion, $\mathbf{Y}_\infty$, equals the simple average of the group members' initial opinions.

(20) $\mathbf{Y}_\infty = \mathbf{W}^\infty \mathbf{Y}_1$

The mutual influence model closely resembles Friedkin and Cook's dynamic model. The β in

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31 Actually, they specify α as being equal to $\delta(1 - \beta)$, where δ represents the effect of forces exogenous to the group that move the members' opinion toward some extreme position. I leave this parameter out of the expression above because it equals one in the absence of such exogenous forces, the situation that obtained in the case of Friedkin and Cook's data and, I expect, that would typically obtain in family data if not in all multiple respondent data.

Friedkin and Cooks' model corresponds to the β's in the mutual influence model, and the Γ_0 corresponds roughly to the Γ's in the mutual influence model. The differences between the two models are: (a) Friedkin and Cook's inclusion of the weight matrix **W** to take the group's communication structure into account and (b) the fact that Γ_0 in the Friedkin and Cook specification refers to the effect of the exogenous variables on initial opinions, while the Γ's in the mutual influence model refer to the effects of the exogenous variables on settled opinions. Although a formal reconciliation of the two models lies outside the scope of this paper, these differences seem small.

The status of the family factor model as the limiting condition of the mutual influence model under consensus is apparent in the correspondence between the features of Friedkin and Cook's model at the limit and the implications of the family factor model for the parameters of the mutual influence model. First, the β's of one in the mutual influence model correspond to Friedkin and Cook's specification that the β in their model approaches one as the group approaches complete consensus. Second, the zero Γ's in the mutual influence model correspond to Friedkin and Cook's specification that the influence of the exogenous variables decreases as the interpersonal influence increases. Third, the zero variance of the errors in equations is consistent with Friedkin and Cook's interpretation of consensus as the limiting condition of their model.

Equation 20 more directly connects the limit of Friedkin and Cook's model to the family factor model. In discussing equation 20, they note that the equality of each member's settled opinion to the mean intial opinion of the group justifies the widespread practice of using the mean settled opinion of a group to represent interpersonal or contextual effects (Friedkin and Cook, 1990:133). As Hauser (1988) shows, however, the use of the mean settled opinion also is a precursor to the family factor model. In other words, we can interpret equation 20 as an early specification of the family factor model. The difference between the mutual influence and family factor models, then, turns on the degree of interpersonal influence in the group (β). When the interpersonal influence is less than one, the mutual influence is needed to represent the relations among the members' opinions. As the group approaches consensus, the mutual influence model evolves into a family

factor model. The substantive and theoretical value of Friedkin and Cook's model, supported by experimental data, is that it provides a plausible interpretation for family factor models when they occur.[32]

This interpretation raises the question of why the family achieves consensus with respect to the student's post-secondary educational plans, while the couple fails to achieve consensus with respect to the plans for another child. Although a definitive answer to this question lies outside the scope of this paper, three answers are possible: First, it is possible that a mutual influence model better represents the relations among the student's, mother's, and father's respective plans; the family factor does better because the data do not include the instrumental variables needed to identify the reciprocal effects. Second, the Jennings-Niemi sample is more homogeneous with respect to the parameters that connect the variables to one another. The Thomson-Williams sample could contain a heterogenous set of couples that range from those for whom a family factor model would hold to those who have no plans and/or are unaware of their spouse's plans if they do exist. Finally, the family, in fact, may be better because the family plans have coalesced after at least twelve years of experience with the educational system.

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32 In light of this intepretation, Hauser and Wong's (1989) model of the educational attainment of siblings which uses both reciprocal effects and a family factor to explain the correlation between siblings' schooling seems misleading. Hauser and Wong use the siblings' respective intellectual abilities, which have different effects on schooling of the person and his sibling, to identify the reciprocal effects; they use father's schooling, mother's schooling, and father's occupation, which affect both siblings' schooling equally to identify the family education factor. Formally, however, Hauser and Wong's model is equivalent to a mutual influence model in which the ability variables identify the reciprocal effects, the socioeconmic variables (equally) affect both siblings' schooling, and the errors in equations for the two siblings' schooling covary positively. What is gained by the specification of a family factor in this case? If we accept the interpretation of the family factor model as the mutual influence model under complete consensus, the existence of one implies the absence of the other. Without good substantive reasons for doing otherwise, therefore, it seems wise to keep the two models separate.

APPENDIX A

STRUCTURAL EQUATION MODELS FOR THOMSON-WILLIAMS DATA

Figure 1. Reduced Form: Mutual Influence and Family Factor Models

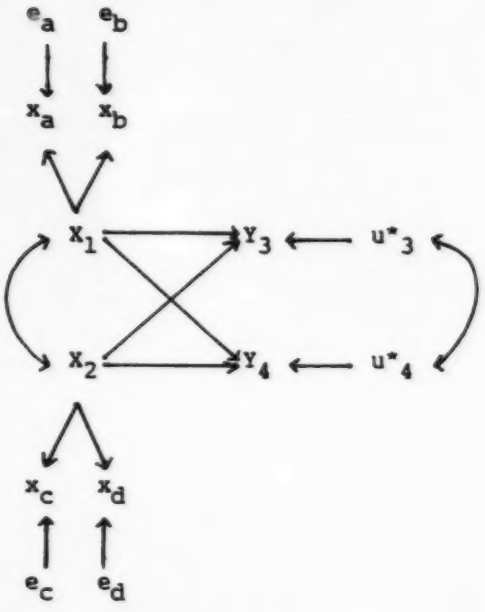


Figure 2. Mutual Influence Model

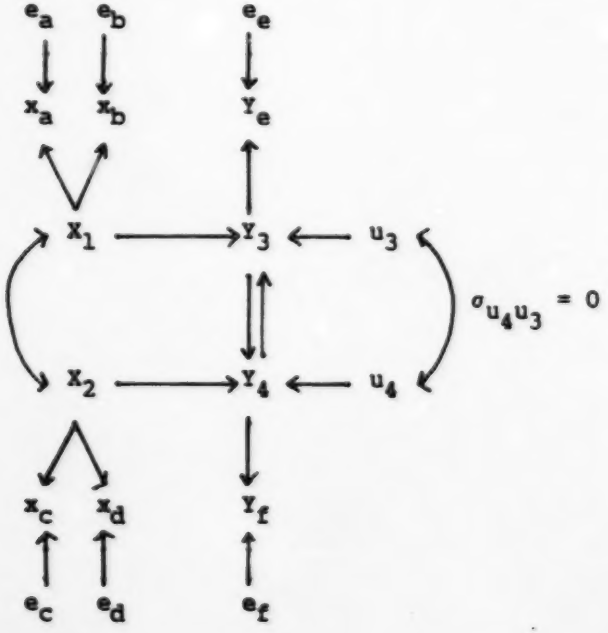


Figure 3. Family Factor Model

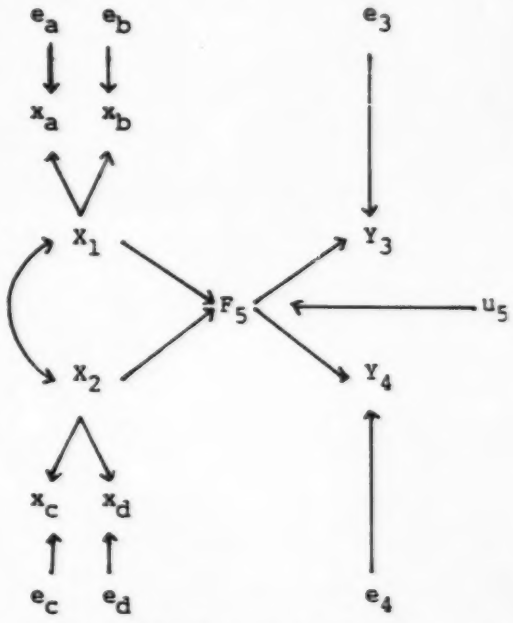
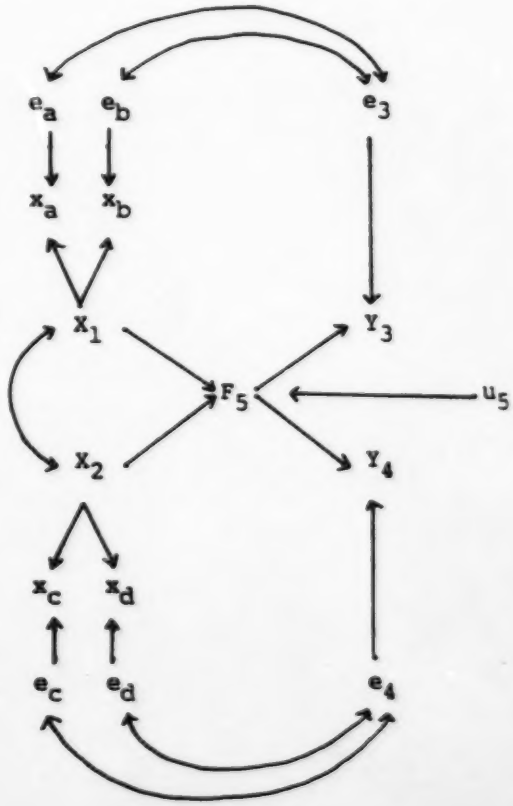


Figure 4. Family Factor Model with Correlated Disturbances



APPENDIX A

STRUCTURAL EQUATION MODELS FOR JENNINGS-NIEMI DATA

Figure 5. Reduced Form

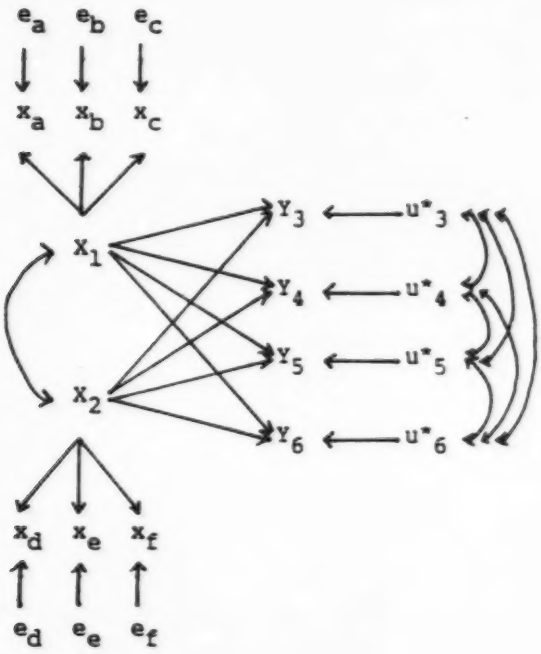


Figure 6. Mutual Influence Model

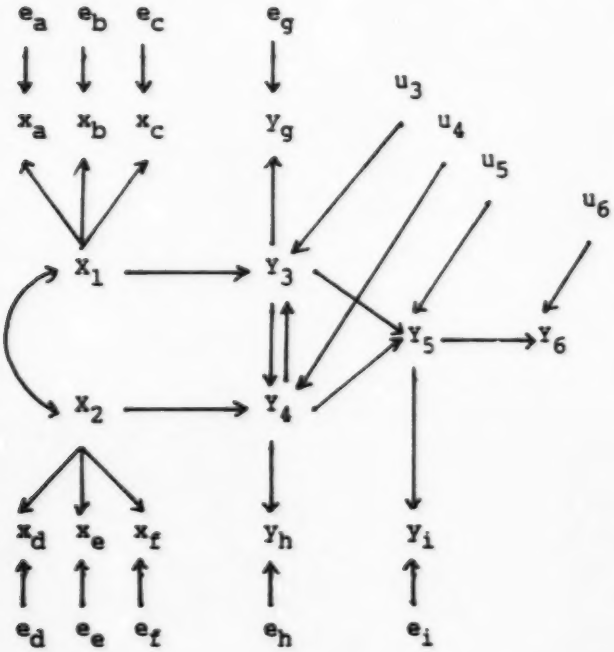


Figure 7. Family Factor Model

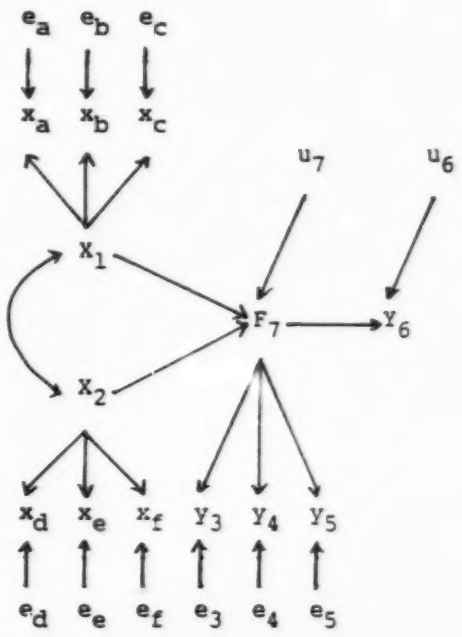
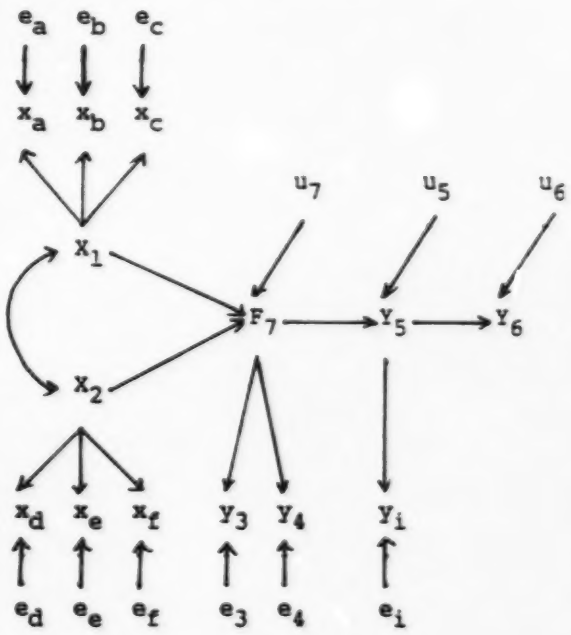


Figure 8. Couple Factor Model



APPENDIX B
CALCULATING THE VARIANCES OF THE DISTURBANCES IN A MUTUAL INFLUENCE MODEL

Assume without loss of generality that the variables in equations 1a and 1b are expressed as mean deviates. Squaring both equations, taking expectations, and solving for the variances of the reduced-form disturbances, $\sigma_{u^*_3u^*_3}$ and $\sigma_{u^*_4u^*_4}$, we get:

$$\text{(B1a)}\ \sigma_{u^*_3u^*_3} = \sigma_{33} - b^2_{31}\sigma_{11} - 2b_{31}b_{32}\sigma_{21} - b^2_{32}\sigma_{22}$$

$$\text{(B1b)}\ \sigma_{u^*_4u^*_4} = \sigma_{44} - b^2_{41}\sigma_{11} - 2b_{41}b_{42}\sigma_{21} - b^2_{42}\sigma_{22}$$

Using equations 3a and 3b (in the text), we can write the variances of the reduced-form disturbances in terms of the variances and covariance of the disturbances in the structural equations of the mutual influence model.

$$\text{(B2a)}\ \sigma_{u^*_3u^*_3} = \frac{\sigma_{u_3u_3} + 2\beta_{34}\sigma_{u_4u_3} + \beta^2_{34}\sigma_{u_4u_4}}{(1 - \beta_{34}\beta_{43})^2}$$

$$\text{(B2b)}\ \sigma_{u^*_4u^*_4} = \frac{\beta^2_{43}\sigma_{u_3u_3} + 2\beta_{43}\sigma_{u_4u_3} + \sigma_{u_4u_4}}{(1 - \beta_{34}\beta_{43})^2}$$

If we assume that the covariance between the disturbances $\sigma_{u_4u_3}$ equals 0 and solve for $\sigma_{u_3u_3}$ in equation B2a and $\sigma_{u_4u_4}$ in equation B2b, we get:

$$\text{(B3a)}\ \sigma_{u_3u_3} = (1 - \beta_{34}\beta_{43})^2\sigma_{u^*_3u^*_3} - \beta^2_{34}\sigma_{u_4u_4}$$

$$\text{(B3b)}\ \sigma_{u_4u_4} = (1 - \beta_{34}\beta_{43})^2\sigma_{u^*_3u^*_3} - \beta^2_{43}\sigma_{u_3u_3}$$

Next, substitute the right-hand side of B3b for $\sigma_{u_4u_4}$ in equation B3a and solve for $\sigma_{u_3u_3}$. Then substitute the right-hand side of B3a for $\sigma_{u_3u_3}$ in B3b and solve for $\sigma_{u_4u_4}$. The result is equations B4a and B4b.

$$(B4a)\ \sigma_{u_3u_3}(1 - \beta_{34}^2\beta_{43}^2) = (1 - \beta_{34}\beta_{43})^2(\sigma_{u^*_3u^*_3} - \beta_{34}^2\sigma_{u^*_4u^*_4})$$

$$(B4b)\ \sigma_{u_4u_4}(1 - \beta_{34}^2\beta_{43}^2) = (1 - \beta_{34}\beta_{43})^2(\beta_{43}^2\sigma_{u^*_3u^*_3} - \sigma_{u^*_4u^*_4})$$

Since $1 - \beta_{34}^2\beta_{43}^2$ equals the product $(1 - \beta_{34}\beta_{43})(1 + \beta_{34}\beta_{43})$, equations B4a and B4b reduce to B5a and B5b, which are given as equations 13a and 13b in the text.

$$(B5a)\ \sigma_{u_3u_3} = (1 - \beta_{34}\beta_{43})\frac{\sigma_{u^*_3u^*_3} - \beta_{34}^2\sigma_{u^*_4u^*_4}}{1 + \beta_{34}\beta_{43}}$$

$$(B5b)\ \sigma_{u_4u_4} = (1 - \beta_{34}\beta_{43})\frac{\beta_{43}^2\sigma_{u^*_3u^*_3} - \sigma_{u^*_4u^*_4}}{1 + \beta_{34}\beta_{43}}$$

APPENDIX C
CALCULATING THE VARIANCES OF THE ERRORS IN VARIABLES IN A MUTUAL INFLUENCE MODEL

Duncan (1975:115–118) shows that estimation of the variance in the random error of a single measure is possible in a three-variable model under the following conditions: First, the effect of the antecedent variable on the ultimate dependent variable is totally indirect. Second, the latent variable is the intervening variable. Third, the disturbance associated with the latent variable is independent of the disturbance associated with dependent variable. The first two conditions are satisfied in estimating the measurement error of wives' and husbands' plans in a mutual influence model because the cross-person effect of one person's taste for children on his or her spouse's plans is indirect. Setting the covariance between the disturbances for plans equal to zero satisfies Duncan's third condition. I derive these results below, thereby extending Duncan's solution to a four-variable model with reciprocal causation.

Using equations 1a and 1b (in the text), we can rewrite the equations 14e and 14f for the measures of wife's and husband's plans, y_e and y_f:

$$(C1a)\ y_e = b_{31}x_1 + b_{32}x_2 + u^*_3 + e_e$$

$$(C1b)\ y_f = b_{41}x_1 + b_{42}x_2 + u^*_4 + e_f$$

Assume without loss of generality that the variables in C1a and C1b are expressed as mean deviates. Squaring both equations, taking expectations, and solving for the two disturbance variances in each equation reproduces equations B1a – B2a in Appendix B with the addition of the measurement error variances.

$$\text{(C2a)}\ \sigma_{u^*_3 u^*_3} + \sigma_{e_e e_e} = \sigma_{y_e y_e} - b^2_{31}\sigma_{11} - 2b_{31}b_{32}\sigma_{21} - b^2_{32}\sigma_{22}$$

$$\text{(C2b)}\ \sigma_{u^*_4 u^*_4} + \sigma_{e_f e_f} = \sigma_{y_f y_f} - b^2_{41}\sigma_{11} - 2b_{41}b_{42}\sigma_{21} - b^2_{42}\sigma_{22}$$

Using equations 3a and 3b (in the text), we can rewrite the left-hand side of equations C2a and C2b in terms of variances and covariance of the disturbances in the structural equations of the mutual influence model.

$$\text{(C3a)}\ \sigma_{u^*_3 u^*_3} + \sigma_{e_e e_e} = \frac{\sigma_{u_3 u_3} + 2\beta_{34}\sigma_{u_4 u_3} + \beta^2_{34}\sigma_{u_4 u_4}}{(1 - \beta_{34}\beta_{43})^2} + \sigma_{e_e e_e}$$

$$\text{(C3b)}\ \sigma_{u^*_4 4^*_4} + \sigma_{e_f e_f} = \frac{\beta^2_{43}\sigma_{u_3 u_3} + 2\beta_{43}\sigma_{u_4 u_3} + \sigma_{u_4 u_4}}{(1 - \beta_{34}\beta_{43})^2} + \sigma_{e_f e_f}$$

To calculate the variances of the measurement error terms in y_e and y_f, we also need an expression for the covariance between y_f and y_e. We get this result by multiplying equations C1a and C1b and taking expectations. Solving for the covariance between the reduced-form disturbances yields equation C4a. Rewriting the covariance between the reduced-form disturbances in terms of the variances and covariance of the disturbances in the structural equations of the mutual influence model yields C4b. (See equations 3a and 3b in the text.) Note that this result relies on the usual assumption that measurement errors are uncorrelated ($\sigma_{e_f e_e} = 0$).

$$\text{(C4a)}\ \sigma_{u^*_4 u^*_3} = \sigma_{y_f y_e} - b_{31}b_{41}\sigma_{11} - (b_{31}b_{42} - b_{41}b_{32})\sigma_{21} - b_{42}b_{32}\sigma_{22}$$

$$\text{(C4b)}\ \sigma_{u^*_4 u^*_3} = \frac{\beta_{43}\sigma_{u_3 u_3} + (\beta_{34} + \beta_{43})\sigma_{u_4 u_3} + \beta_{34}\sigma_{u_4 u_4}}{(1 - \beta_{34}\beta_{43})^2}?$$

Next, we assume that the covariance between the disturbances, $\sigma_{u_4u_3}$, equals zero. Further, we assume that the variance of u_3 equals the variance of u_4: $\sigma_{u_3u_3} = \sigma_{u_4u_4} = \sigma_{uu}$. Finally, although not necessary, we assume the variances of the measurement error terms e_e and e_f are equal: $\sigma_{e_ee_e} = \sigma_{e_fe_f} = \sigma_{ee}$. With these assumptions, equations C3a, C3b, and C4b reduce to equations C5a – C5c.

$$\text{(C5a)} \quad \sigma_{u^*_3u^*_3} + \sigma_{e_ee_e} = \frac{\sigma_{uu}(1 + \beta^2_{34})}{(1 - \beta_{34}\beta_{43})^2} + \sigma_{ee}$$

$$\text{(C5b)} \quad \sigma_{u^*_4u^*_4} + \sigma_{e_fe_f} = \frac{\sigma_{uu}(1 + \beta^2_{43})}{(1 - \beta_{34}\beta_{43})^2} + \sigma_{ee}$$

$$\text{(C5c)} \quad \sigma_{u^*_4u^*_3} = \frac{\sigma_{uu}(\beta_{43} + \beta_{34})}{(1 - \beta_{34}\beta_{43})^2}$$

We can use C5c to solve for σ_{uu}. Substituting this result for σ_{uu} in either equation C5a or C5b allows us to solve for σ_{ee}.

APPENDIX D
PROCEDURES USED IN ANALYZING THE JENNINGS-NIEMI DATA

The Jennings-Niemi data come from two data sets made available through ICPSR: the 1965 survey of parents and students (ICPSR, 1971) and the second wave of student interviews conducted in 1973 (ICPSR, 1981). The bulk of the data analyzed herein come from the 1965 survey. The data come in the form of one long record per case of student information followed by parent information. In the triads subsample, there are two student-parent records per family. The first record contains parent the information from the mother; the second contains the parent information from the father and duplicates the information from the student. In setting up the data for analysis, I used a parent identification variable to write the variables for the analysis into separate files of student-mother dyads, student-father dyads, and triads. Since the records for each triad are adjacent to one another, I could define the triads in subsequent analyses by specifying two records per case in the format statement for the triads file. As a check on the format

statement, I correlated the two sets of students variables with one another. Since one set just duplicates the other, the two sets should correlate perfectly; I obtained these results. I used the parent identification variable to compute variables in the triads file for the mother and father, respectively. This procedure guarantees that the variables were assigned to the right parent. To complete the data set, I used a student identification variable common to both waves to merge the data in the three 1965 files with data on the student's schooling in the 1973 student interviews.

All the variables required extensive recoding to convert them into years of school. To obtain the student's schooling, I assigned a 12 to students who said they had stopped after high school, a 13 to students who reported post-secondary education but no college, and the numbers 13 through 20 to students on the basis of the years of college reported by the students who did attend. In recoding the students plans, I assigned a 12 to those students who said they would not continue their education after high school; I assigned a 12 to parents who mentioned something other than education in response to a question about what the parent hoped the student would do in the year following high school graduation. I assigned a 14 to both students and parents who reported some form of junior college in response to a followup question to the initial plans question; I assigned a 16 to respondents who mentioned a four-year program; and I assigned a 13 to the remainder of valid responses. I used the number of years of schooling to recode the responses to the questions about the parents' schooling. In the case of reports of less than high school but some post-secondary training (e.g., beauty school), I simply assigned the number of years of high school attended and did not attempt to convert the post-secondary training into additional years of schooling. Finally, I assigned 18 to all reports of postgraduate schooling. Clearly, additional experimentation with different coding schemes is warranted in order to test the robustness of the findings reported in the text.

The procedure for incorporating the dyads into the parameter estimation involved the simultaneous analysis of the correlations, standard deviations, and means for the three groups. In preparing the matrices for the dyads subsamples, I assigned a "pseudo value" of one to the standard deviations of the missing variables and a pseudo value of zero to the correlations and means.

These matrices are reproduced in Tables D.1 and and D.2 below. In estimating the models, I set the variance of the errors in variables (the θ_{ee}'s) for the missing variables equal to one and the means (the τ_y's) for these variables equal to zero.

Because of the addition of pseudo values to the matrices, LISREL incorrectly calculates the degrees of freedom. Instead of one subsample of 10 means, 10 standard deviations, and 45 correlations plus two subsamples of 7 means, 7 standard deviations, and 21 correlations, LISREL bases the calculation of degrees of freedom on three subsamples of 10 means, 10 standard deviations, and 45 correlations. Thus, the correct number of degrees of freedom is 60 less than the number reported in the LISREL output. In order to obtain the correct p-value for the chi-square and right degrees of freedom, I used the computation routine of SST (Dubin and Rivers, 1986).

Table D.1
Correlations, Standard Deviations, and Means
for 269 Student-Mother Dyads

	Variables									
	SED	SPLN	MPLN	FPLN	SFED	MFED	FFED	SMED	MMED	FMED
SED	1.00									
SPLN	.533	1.00								
MPLN	.573	.770	1.00							
FPLN	0.00	0.00	0.00	1.00						
SFED	.353	.325	.319	0.00	1.00					
MFED	.381	.314	.307	0.00	.882	1.00				
FFED	0.00	0.00	0.00	0.00	0.00	0.00	1.00			
SMED	.325	.234	.208	0.00	.540	.540	0.00	1.00		
MMED	.370	.284	.259	0.00	.552	.572	0.00	.841	1.00	
FMED	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
SD's	1.85	1.53	1.61	1.00	3.26	3.24	1.00	2.33	2.51	1.00
Means	14.2	14.4	14.3	0.00	11.8	11.8	0.00	11.8	11.7	0.00

In setting up the LISREL models, I treated all variables as y variables in order to allow the modification indexes to test for systematic measurement error across the measures of the exogenous and endogenous variables. In the this search I looked for measures that loaded positively on constructs other than the ones they measured (for example, a positive effect of father's school-

Table D.2
Correlations, Standard Deviations, and Means
for 258 Student-Father Dyads

	Variables									
	SED	SPLN	MPLN	FPLN	SFED	MFED	FFED	SMED	MMED	FMED
SED	1.00									
SPLN	.603	1.00								
MPLN	0.00	0.00	1.00							
FPLN	.598	.710	0.00	1.00						
SFED	.432	.333	0.00	.363	1.00					
MFED	0.00	0.00	0.00	0.0	0.00	1.00				
FFED	.440	.369	0.00	.374	.910	0.00	1.00			
SMED	.391	.302	0.00	.305	.568	0.00	.566	1.00		
MMED	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	1.00	
FMED	.400	.313	0.00	.334	.581	0.00	.610	.912	0.00	1.00
SD's	1.91	1.53	1.00	1.61	3.24	1.00	3.18	2.39	1.00	2.45
Means	14.3	14.4	0.00	14.2	11.3	0.00	11.5	11.8	0.00	11.9

ing on father's report of mother's schooling) and for positive covariances between the errors in reports made by the same person (for example, student's respective reports of father's and mother's schooling). I did not find any evidence of this type of systematic measurement error. The maximum modification index in all four models pertains to the covariance between the errors in variables for the student's and father's reports of father's schooling in the triads subsample. Although freeing this parameter would yield a significant improvement in fit, I retain the specification of zero covariance because the reports come different members and because the information from the estimated change suggests that the covariance would be negatively rather than positively signed.

The only modifications I made involve relaxing some of the equality constraints. The first equality constraint concerns the loadings of the student's plans and the student's reports of his or her parents' respective schooling. I initially set these loadings equal to one (along with the each parent's report), but the modification indexes suggested freeing these loadings would result in a significant improvement in the fit of the model. The second equality constraints involved the means. Although the members' reports of each parent's schooling have the same means, stu-

dent's mean plans are significantly greater than the means of father's and mother's respective plans. The final set of equality constraints concern the cross-subsample constraints discussed below.

I began the estimation by imposing cross-subsample constraints on all relevant parameters (excluding the θ_{ee}'s and τ_y's for the missing variables). Because the subsamples were randomly assigned, the parent variables missing from the dyads data should, in principle, be missing completely at random. In practice, however, the dyads also contain student-parent pairs from triads in which interviews were not obtained from one of the parents and student-parent pairs in which one parent was substituted for the other; thus there is no guarantee that this condition will be met. Nevertheless, the results for all models showed that these constraints fit the data in the case of the means, the slopes, the variance-covariance matrix for the latent variables, and most of measurement error variances. The exceptions occurred in the case of error variance in the student's report of father's schooling, mother's report of father's schooling, and mother's report of her own schooling. The error variance in the student's report of father's schooling was significantly less for students in the triad subsample than for students in both the student-mother and student-father subsamples. The error variance in both the mother's report of father's and her own schooling was significantly less for mothers in the triads subsample than for mothers in the student-mother subsamples.

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